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Stochastic analysis and prioritization of the influence of parameter uncertainty on the predicted pressure profile in heterogeneous, unsaturated soils

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Abstract

This article utilizes a Monte Carlo stochastic framework to investigate the influence on the mean and variance of the predicted mean pressure head profile of statistical assumptions regarding the parameters that enter the mathematical description of the problem of infiltration in unsaturated, heterogeneous layers. The parameters are treated as random functions with an exponential auto-covariance function expressing their spatial continuity. Four different truncated distributions are taken to describe the parameters according to field observations and various phases of site characterization campaigns. The exponential distribution is seen to produce the largest (in absolute value) mean and variance in the pressure head profile. For all distributions the variance in pressure head increases with increasing mean pressure. A second topic of this article is to investigate, the relative importance of each parameter, in terms of the mean and the variance of the predicted pressure. For uniformly or triangularly distributed parameters the saturated hydraulic conductivity appears to dominate the mean-behavior and the uncertainty in the system's solution. For lognormally or exponentially distributed parameters another parameter, the van Genuchten pore-size distribution index, is the dominant factor.

Keywords: Unsaturated flow; Heterogeneous soils; Monte Carlo simulations; Parameter ranking

1. Introduction

One of the major tasks facing hazardous waste projects is to assess the critical components that influence the performance of a system and which affect predictive capabilities. Prioritization of the critical factors whose uncertainty reduction narrows a system's output uncertainty is important at both the site characterization and the modeling level. Such prioritization allows the concentration of human and monetary resources on only those factors that critically influence a system's predicted performance. The objective of this work is to illustrate, through the analysis of a physical problem of unsaturated flow in stratified, heterogeneous media, how such concepts can be utilized to guide site characterization and modeling efforts. The focus is the quantification of the effect of assumptions about the statistical structure of data on the prediction of pressure head profile as well as the evaluation of the relative importance of three physical parameters that enter the description of the problem. Field data from six hydrogeologic units at the US Department of Energy repository site of radioactive wastes at Yucca Mountain, Nevada, were utilized. The physical problem is characterized by three parameters exhibiting both spatial variability and uncertainty in their values: the K_s, the saturated hydraulic conductivity; the α, the van Genuchten air-entry scaling parameter; and β the van Genuchten pore-size distribution index parameter that enter into the governing nonlinear equations. The main objective of this study is to investigate the effect of the parameters' probability distribution functions on the prediction of the pressure head profile. This issue arises from the limitation in data for many earth studies, which as a consequence allows several probability distribution models to fit the data. In addition, for large projects where several site characterization campaigns are conducted over time, the statistical description of the data at each distinct site characterization phase may vary and hence the modeling efforts (and predictions) that correspond to these data collection phases may rely on different or evolving assumptions. The ranking of the three physical parameters relative to their influence on the mean and variance of pressure head is the second focal point of this article.

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The article proceeds as follows. Section 2 provides a literature review of the sensitivity, and uncertainty methodologies as well as of the measures that have been used to evaluate a system's components and their influence on its performance. Section 3 discusses the physical problem and provides the mathematical and numerical framework of the study. This section also presents the sensitivity and uncertainty analysis and the relative importance measures that were used to rank each parameter towards the predicted pressure head profile. Finally, Section 4 presents the results and conclusions of this study.

2. Background

In broad terms several distinct analyses can be conducted at hazardous waste sites. *Uncertainty analyses* involve determining the uncertainty in model predictions, which results from imprecisely known variables or parameters. *Sensitivity analyses* are used to study the behavior of a system, or a model, and ascertain how much the outputs depend on each of the input parameters. *Importance analyses*' measures are usually applied to systems whose functionality is modeled as success or failure and whose components are modeled in one of two binary states. *Decision analyses* identify the best alternative from a suite of available alternatives. The main elements of decision analyses are the action space that contains all available alternatives, the state of nature that is governed by uncertainty (for example, uncertainties in the estimates of transmissivity, storativity, etc.), and the consequences of different actions, which includes benefits and costs. The distinction between the different types of analysis in most studies is not rigid, and several authors have utilized or developed approaches that encompass elements from more than one type of analysis. For example, Ma [1] combined Monte Carlo simulation to propagate parameter uncertainty and elements of sensitivity and importance analyses to reach remediation decisions at a groundwater site in Taiwan. The methodology utilized in the current study proceeds in this spirit by combining Monte Carlo simulations with elements of uncertainty, sensitivity and relative importance analyses.

2.1. Sensitivity analyses

A number of ranking measures have been used in several scientific and engineering fields and can be tested for their applicability in surface–subsurface hydrologic systems. Mansour and Wirsching [2] provided sensitivity and reliability indices appropriate for strength and fatigue considerations in extreme loading conditions on ships. These authors provided an overview of sensitivity factors in structural reliability methods. In general, a deterministic sensitivity factor S_i of a response variable R that is a function of a vector, Y, of design factors Y_i, $i = 1, \ldots, n$, is defined as the change in R with respect to a change in Y at a reference value $Y = Y_0$. One can consider also probabilistic sensitivity factors for a response variable $R = R(Y)$, where Y is now a random vector. A reliability-based sensitivity factor, which measures the degree of response uncertainty as a function of input uncertainty, is commonly obtained by evaluating the change in the standard deviation of R with respect to the standard deviation of Y_i. Certain other sensitivity factors obtained from first- and second-order reliability methods have proceeded with the transformation of the coordinates of a design vector Y into a normal space U and using minimum distance concepts to define a safety index β, as the minimum distance from the origin to the limit state function in u-space. Importance factors and sensitivity of the cumulative distribution function of a response R to a statistical parameter (e.g. mean or standard deviation) are defined on the transformed space.

Homma and Saltelli [3] developed a global sensitivity analysis (SA) method of nonlinear models that is based on measures of importance that account for the fractional contribution of input parameters to the variance of the model prediction. Global SA focuses on output uncertainty over the entire range of values of input parameters. These authors' measures of importance were based on the conditional variance of model output that corresponds to parameters set at fixed values. Good reviews of global SA methods, which include the Monte Carlo based regression–correlation measures, the Fourier amplitude sensitivity test (FAST), and various forms of differential analysis can be found in Helton [4]. The concept of regional sensitivity analysis (RSA) can be summarized as follows [5]: a model G contains a set of constant parameters x and a set of inputs z which together produce an output y. Uncertainty and/or variability in the parameters is described by assigning to each element of x a statistical distribution function which, taken together, constitute a multivariate distribution function f. This assignment results in an ensemble of models, each of structure G, and with a parameter vector, which is a random number of the multivariate distribution f. The goal is to ascertain which elements of the parameter vector are important in producing simulations that mimic the essential features of the system.

2.2. Importance analyses

For ranking basic events there are several commonly used importance measures [6]. If one designates by R_0 the base (reference) case overall model risk, by R_i^+ the overall model risk with probability of a basic event set to 1 (the event has occurred or the equipment has failed), and by R_i^- the overall model risk with the probability of basic event set to 0 (the event is impossible or the equipment is totally reliable) then several measures can be utilized.

2.2.1. Risk achievement worth

$a_i = R_i^+/R_0$, presents a measure of the worth of a basic event in achieving the present level of risk and indicates the importance of maintaining the current level of reliability for a basic event.

2.2.2. Risk reduction worth

$r_i = R_0/R_i^-$, represents the maximum decrease in risk by an improvement to an element associated with a basic event (i.e. useful for identifying improvements to the reliability of elements, which mostly reduce risk).

2.2.3. Fussell–Vesely

$\mathrm{FV}_i = (R_0 - R_i^-)R_0 = 1 - (R_i^-/R_0)$, is a measure of the fractional contribution of the basic event to the overall model risk when the basic event probability is changed from its base value to 0.

2.2.4. Birnbaum Importance (or reliability importance)

$I_\mathrm{B} = R_i^+ - R_i^-$, is an interval risk importance measure, which is completely dependent on the structure of the system model and is independent of the current probability of the basic event.

2.2.5. Criticality importance

$I_\mathrm{cr} = (R_i^+ - R_i^-)P_{i,0}/R_0 = I_\mathrm{B} P_{i,0}/R_0$, where, $P_{i,0}$ = probability of basic event I at its reference value.

Traditionally, importance measures are based on frequency of failure. Eisenberg and Sagar [6] proposed measures intended to be more suitable to systems comprised of components whose behavior is most naturally represented as continuous rather than binary. These authors considered systems whose performance measure, Y, is positive and increases with poorer performance and where regulatory standards limit the magnitude of Y. For X_K random input parameters that are described by appropriate probability distributions, $Y = Y(X_\mathrm{K})$ is a random variable. If the nominal system performance is S_y and the system performance where a component is neutralized is $-U_y$, where the convention "$-U$" indicates that the component functions are to be neutralized, these authors considered four importance measures that were ratios of $-U_y$ and S_y.

2.3. Uncertainty analyses

Protopapas and Bras [7] and Ricotti and Zio [8] investigated input uncertainty that refers to incomplete knowledge of model inputs, including model parameters. Suppose that for a model m the prediction y is determined by a vector of input variables x of length p (or parameter vector θ), $y = m(x)$. The probability distribution f_x of the input variables x induces on y the probability distribution f_y, i.e., $x \sim f_x(x)$, $y \sim f_y(y)$. A subset of input variables is x^s (control variables) and $x^{\bar{\mathrm{s}}}$ is the complementary input subset variables (noise variables).

The model prediction based on subset x^s of the input variables is: $\tilde{y} = E(y|x^\mathrm{s})$. The model prediction based on x^s and $x^{\bar{\mathrm{s}}}$ is: $y = \tilde{y} + (y - \tilde{y}) = E(y|x^\mathrm{s}) + e(x^{\bar{\mathrm{s}}}|x^\mathrm{s})$, where the first term represents the average fixed value we expect for y due to the control variables x^s and the second term represents the random residual or error component due to the noise variables $x^{\bar{\mathrm{s}}}$. The authors' objective was to find a small subset of the x^s for which their predictor $\tilde{y}$ is a good approximation to the full model prediction y. A way to measure the quality of a predictor is the quadratic loss function: $L = (y - \tilde{y})^2$, with $E(L)$ the mean squared error (MSE) of the prediction. Their method relates the importance of the set x^s to its predictive ability (locally at a specified value of x^s) by the L and globally averaged over values of x^s by the $E(L)$.

Zio and Apostolakis [9] applied sensitivity and uncertainty analysis techniques to evaluate environmental restoration technologies. Their methodology integrated several standard decision analysis tools such as Influence Diagrams (IDs), Analytical Hierarchy Process (AHP), and Multi Attribute Utility Theory (MAUT). They used a global performance index, which was a weighted combination of the utility functions of the individual performance measures. They assessed the weights through the hierarchical structure of the ID and by means of the pairwise comparison method of the AHP. These weights represented the direct importance of the performance measures with respect to the overall top goal of the hierarchy, and were obtained by multiplying the intermediate weights through the various branches of the hierarchy tree. For the determination of the single-attribute utilities these authors employed AHP combined with elements of fuzzy logic.

Weil and Apostolakis [10] developed a prioritization methodology that utilized multi-attribute utility theory with emphasis on deliberations to achieve consensus among objectives and preferences among those objectives. They assigned a numerical performance index to each item requiring prioritization. The item with the highest numerical score received the highest priority. The performance index (PI) was the sum of the weights of individual performance measures (PM) multiplied by the utilities of each item for that particular PM. The purpose of the performance measures was to relate some measurable quantity, representing an attribute from the item, to the fundamental objectives. Their procedure to calculate the performance index had the following steps: (i) structure the objectives (economical, safety, environment, etc.); (ii) determine appropriate performance measures; (iii) weigh objectives and performance measures; (iv) assess utility functions of performance measures; (v) perform consistency checks; (vi) validate the results.

3. Physical problem

The study considers the physical problem of infiltration in heterogeneous, unsaturated porous layers of a typical cross-section at the US Department of Energy site of radioactive wastes at Yucca Mountain, Nevada. The site was formed by a series of volcanic eruptions that deposited ash and other material that were compressed over time to create layers of tuff. As a result the geologic environment is comprised of a series of heterogeneous welded and non-welded silicic airfall and ash flow tuffs, the cooling and crystallization of which determined their mechanical and hydrologic properties. The geologic formations have been grouped into hydrogeologic units based, largely, on the degree of welding. In general, welded units have low matrix porosities (of the order of 10%) and high fracture densities. Non-welded units have higher matrix porosities (about 30%) and low fracture densities [11]. The hydrogeologic units that are considered in most unsaturated flow analyses as well as in this study are, from top to bottom: the Tiva Canyon welded (TCw) with a thickness of 81 m, the Paintbrush nonwelded (PTn) with a thickness of 39 m, the Topopah Spring welded (TSw) with a thickness of 299 m, the Topopah Spring vitrophyre (TSv) with a thickness of 15 m, the Calico Hills nonwelded-vitric (CHnv) with a thickness of 64 m, and the Calico Hills nonwelded-zeolitic (CHnz) with a thickness of 127 m [12,13]. The units exhibit significant differences in their properties [14] and hydraulic behaviors, and data that were available for this study included the thickness of layers, the means and standard deviations, and the range of the parameters.

3.1. Conceptual model

Assuming, as is commonly done [14–17], that the infiltration rate is constant at the site, and that steady-state conditions have been attained, the hydrologic state between the ground surface and the water table (pressure head and saturation against depth) can be described by Richard's equation and the van Genuchten [18] relation for the unsaturated hydraulic conductivity. The specific discharge for one-dimensional, steady state, vertical infiltration is given by:

$$q = -K(h_\mathrm{p})\left(\frac{\mathrm{d}h_\mathrm{p}}{\mathrm{d}z} + 1\right) \tag{1}$$

where h_p is the pressure head (negative when the soil is unsaturated), z is the vertical Cartesian coordinate (positive upward), $K(h_\mathrm{p})$ is the unsaturated hydraulic conductivity (a function of the pressure head), and q is the infiltration. The unsaturated hydraulic conductivity is expressed through the van Genuchten relation [18]

$$K(h_\mathrm{p}) = K_\mathrm{s}(1 + |\alpha h_\mathrm{p}|^\beta)^{-(1-\beta^{-1})/2} \times \left(1 - \left[\frac{|\alpha h_\mathrm{p}|^\beta}{1 + |\alpha h_\mathrm{p}|^\beta}\right]^{1-\beta^{-1}}\right)^2 \tag{2}$$

Here K_s is the saturated hydraulic conductivity, α, is the van Genuchten air-entry scaling parameter; and β is the van Genuchten pore-size distribution index parameter. Eq. (1) can be easily rewritten as

$$\int_{h_{\mathrm{p}0}}^{h_{\mathrm{p}1}} \frac{K(h_\mathrm{p})\,\mathrm{d}h_\mathrm{p}}{(K(h_\mathrm{p}) + q)} = -(z_1 - z_0) = -\Delta z \tag{3}$$

where $h_{\mathrm{p}0}$ corresponds to elevation z_0 (taken to be the water table) and $h_{\mathrm{p}1}$ the pressure at an arbitrary point of elevation z_1. The one-dimensional flow domain was discretized into elements of length 0.1 m for a total of 6250 nodes. The constant infiltration rate was taken to be 0.1 mm/yr, a value that is considered to be representative of the conditions at the site [14]. Eq. (3) was solved numerically, using an iterative procedure that was based on a Newton–Cotes 9-point scheme, as follows. Starting from the water table ($z_0 = 0$ and $h_0 = 0$) the solution of h1 at a distance of $\Delta z = 0.1$ m is sought. An arbitrary value of h_1^0 is selected which together with the van Genuchten relation determines the integral at the left-hand side of Eq. (3). The value of the integral is compared to the right-hand side value of -0.1 m and h_1^0 is corrected by a quantity that depends on the residual error of Eq. (3). The procedure is repeated until a value of h_1 is found such that the residual error of Eq. (3) is smaller than 0.001 m. This value of pressure is the solution for point $z_1 = 0.1$ m and the procedure advances by assigning these values as the new, starting elevation and pressure and seeking to find the solution for pressure at a distance of 0.2 m from the water table. The above numerical procedure was compared with the results by Reeves et al. [19] using the same fixed (deterministic) values for the parameters K_s, α, and β for each layer [20] of the stratigraphy that was used by these authors. Fig. 1 shows an excellent agreement between the results of this study and Reeves et al. [19].

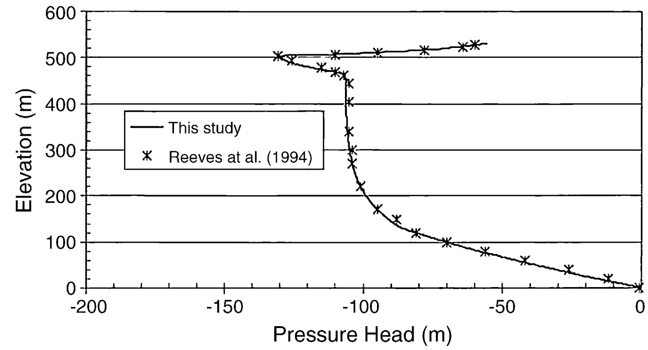

Fig. 1. Comparison of deterministic numerical results (this study) and Reeves et al. [19].

3.2. Modeling of heterogeneity and ranking procedure

Parameters K_s, α, and β were considered in the study as random variables that follow the same probability distribution. The distributions used for the parameters in the analysis were: the lognormal, the exponential, the uniform, and the triangular. The lognormal distribution was chosen because it has been shown to fit data at several sites [21–24]. The exponential distribution is appropriate for cases where low values of a parameter (relative to the mean) are expected to occur more often than larger values. This can be the result of material changes from coarser to finer texture, sands of the same grain size but of a stronger cementation in a part of a system than another, a sand–shale system with higher percentage of shale than sand, etc. The uniform distribution was chosen because it describes the (common) situation where one might have knowledge of only the range within which a parameter lies, and the triangular because it represents the case where, in addition to the minimum and maximum values, one might have information about the most commonly occurring value. The last two cases correspond to the very initial stages of a characterization and modeling study where only geologic and other background information may be available for a site allowing only inference of ranges and likely values of the parameters. The cases of lognormal and exponential correspond to a site characterization phase where detailed data collection has taken place allowing detailed statistical description of the data. One should notice that the following discussion and arguments can also be interpreted in a Bayesian decision-making framework where increase of information may result in changes in the ranking of importance of different uncertain components of a physical system, and hence, in re-evaluation of site characterization and modeling priorities of a study.

Truncated forms of the distributions were used with the minimum and maximum values of the parameters providing the truncation limits of the distributions. The generated distributions had means and variances that equalled the values for mean and variance of the parameters from the available data. The parameters were considered to be spatially correlated following an exponential covariance model with identical correlation lengths λ. Three cases of correlations lengths equal to λ = 1 m, λ = 0.3 m, and λ = 3 m were investigated. These values are considered to be representative of the site and have been used in other studies at the Yucca Mountain project [25]. Data that can allow the determination of cross-correlation functional relations are rarely available and two extreme cases of cross-correlation between K_s, α, and β were studied: (i) uncorrelated parameters, i.e., knowledge of one parameter does not provide information on the other two parameters, and (ii) perfectly correlated parameters, i.e., if one parameter takes a min (or max) value then the other two also take a min (or max) value. The random field generator used to generate values for K_s, α, and β that honor the above statistics and to perform the Monte Carlo simulations was based on the Simulated Annealing Method [26].

By selecting a triplet of values from a specific distribution i for the parameter set (K_s, α, β) for each point of the grid and by performing a series of Monte Carlo computations with N total selections one can create N pressure head h_p-profiles. At each discretization point of the grid with a coordinate z one can then average the N equiprobable values of hp to obtain the mean capillary head $\langle h_p(z)\rangle_i$ and the variance of h_p, σ_i^2, that applies to this point for a specific distribution i:

$$\langle h_p(z)\rangle_i = \frac{1}{N}\sum_{j=1}^{N} h_{p_j}, \qquad \sigma_i^2 = \frac{1}{N-1}\sum_{j=1}^{N}(h_p - \langle h_p(z)\rangle_i)^2 \tag{4}$$

Here ⟨ ⟩ denotes ensemble averaging. Now one can calculate, at each point z, the global mean $\langle h_p(z)\rangle_G$, defined as the arithmetic mean of the expected values from the four distributions, as well as the global variance $\langle\sigma^2\rangle_G$, defined as the arithmetic mean of the variances obtained from each distribution:

$$\langle h_p\rangle_G = \frac{1}{4}\sum_{i=1}^{4}\langle h_p(z)\rangle_i, \qquad \langle\sigma^2\rangle_G = \frac{1}{4}\sum_{i=1}^{4}\sigma_i^2 \tag{5}$$

By calculating the quantity $\langle\rho^2\rangle_T = \frac{1}{4}\sum_{i=1}^{4}(\langle h_p(z)\rangle_i - \langle h_p\rangle_G)^2$ one can obtain at each point the divergence of the distributions means from the global mean. Then the total uncertainty on the global mean $\langle h_p(z)\rangle_G$, can be obtained at each point by $\sigma_T^2 = \langle\rho^2\rangle_T + \langle\sigma^2\rangle_G$. Here $\langle\rho^2\rangle_T$ is a measure of the uncertainty in the mean h_p behavior because of the uncertainty in the type of distribution, and $\langle\sigma^2\rangle_G$ is the average fluctuation around the mean h_p-behavior irrespective of distribution.

Assume now that out of the three parameters K_s, α, and β one holds two at their mean values, and varies the third according to a distribution i. By performing Monte Carlo simulations one can obtain $\langle h_p\rangle_{i,k}$ the mean pressure head, and $\sigma_{i,k}^2$ the variance of the pressure head, due to the fluctuations in the kth random parameter according to an ith distribution. By repeating the procedure for all parameters the relative importance can be evaluated towards the mean pressure head and variance, respectively, of each parameter k for every distribution i:

$$\mathrm{RI}_{i,k}^{\langle h_p\rangle} = \frac{\langle h_p\rangle_{i,k}}{\sum_{k=1}^{3}\langle h_p\rangle_{i,k}}, \qquad \mathrm{RI}_{i,k}^{\sigma^2} = \frac{\sigma_{i,k}^2}{\sum_{k=1}^{3}\sigma_{i,k}^2} \tag{6}$$

This way one can determine for a given range of variation of the parameters, which parameter controls uncertainty in the pressure head. Clearly, this process can be repeated for all distributions i and then, for each parameter k, one can calculate the relative importance toward the mean and variance:

$$\mathrm{RI}_{k}^{\langle h_p\rangle} = \frac{\sum_{i=1}^{4}\langle h_p\rangle_{i,k}}{\sum_{i=1}^{4}\sum_{k=1}^{3}\langle h_p\rangle_{i,k}}, \qquad \mathrm{RI}_{k}^{\sigma^2} = \frac{\sum_{i=1}^{4}\sigma_{i,k}^2}{\sum_{i=1}^{4}\sum_{k=1}^{3}\sigma_{i,k}^2} \tag{7}$$

irrespective of distribution. Thus, the above expressions can provide a ranking of the importance of each parameter in the evaluation of the mean and variance of the pressure head for a specific distribution, and irrespective of the choice of distribution [27].

4. Results and conclusions

The framework detailed in the previous section was used to analyze the influence of controlling parameters in problems of infiltration in heterogeneous soils. The parameters (K_s, α, and β) that enter the van Genuchten expression for the unsaturated hydraulic conductivity, Eq. (2), were modeled as random functions with means, coefficients of variation and ranges obtained from field data. The spatial auto-correlation of the parameters was described by an exponential model and the results for a correlation length equal to λ = 1 m are presented here. The numerical calculations were performed using the Simulated Annealing Generating Method and for each parameter and each distribution 500 Monte Carlo simulations were conducted. The results shown here correspond to absence of cross-correlation between the parameters (case I).

Figs. 2 and 3 plot the mean pressure head profile $\langle h_p\rangle$ and the variance about this profile, respectively, for parameters with a correlation length equal to λ = 1 m. The exponential distribution produced a mean and variance in the pressure-head profile that was significantly larger than of any other statistical model, in all layers. The lognormal distribution produced the smallest (in absolute value) mean pressure head profile in the CHnz, TSw, and TCw layers whereas the uniform distribution produced the smallest (in absolute value) mean pressure head in the remaining layers. These results can be explained by the preference of the exponential model to select lower K_s values than any other distribution, which given the inverse relation between pressure head and K_s leads (all other factors being equal) the exponential model to produce the largest (absolutely) pressure head. It is interesting to note that the results for the variance are perhaps counter-intuitive with regards to the uniform and triangular distributions where because of the lack of knowledge that these distributions imply one might have expected them to produce the largest variances. Figs. 2 and 3 indicate that irrespective of distribution type the variance in pressure head increases with

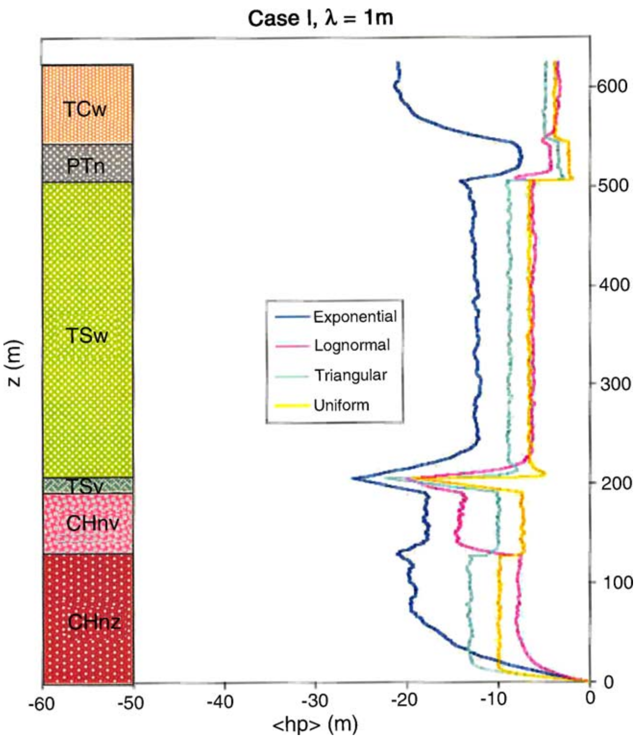

Fig. 2. Mean pressure head profiles (case I, $\lambda = 1$ m): dependence on distribution.

increasing (in absolute terms) mean pressure. This is in agreement with the conclusions by Yeh et al. [28], field observations by Yeh et al. [29], laboratory investigations by Wildenschild and Jensen [30], and the analytical results by Yeh [15]. The dependence of the mean and variance of the pressure-head on the

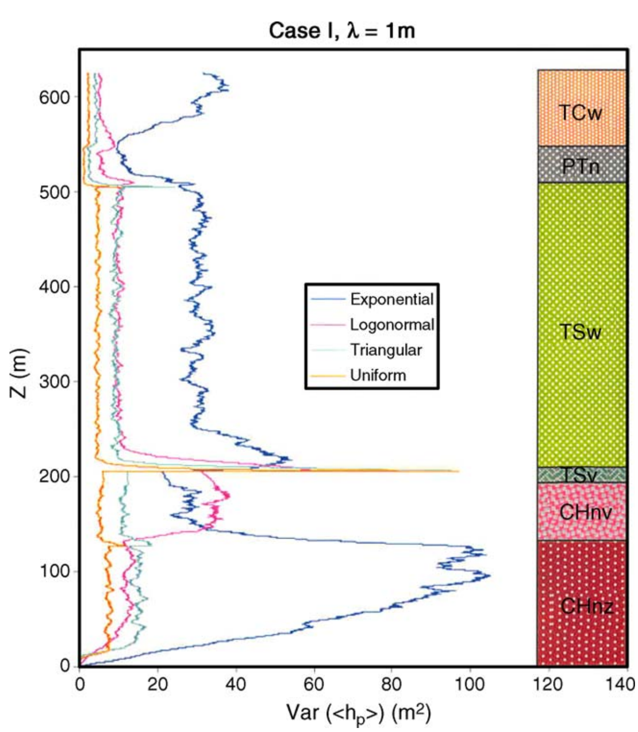

Fig. 3. Variance of pressure head profiles (case I, $\lambda = 1$ m): dependence on distribution.

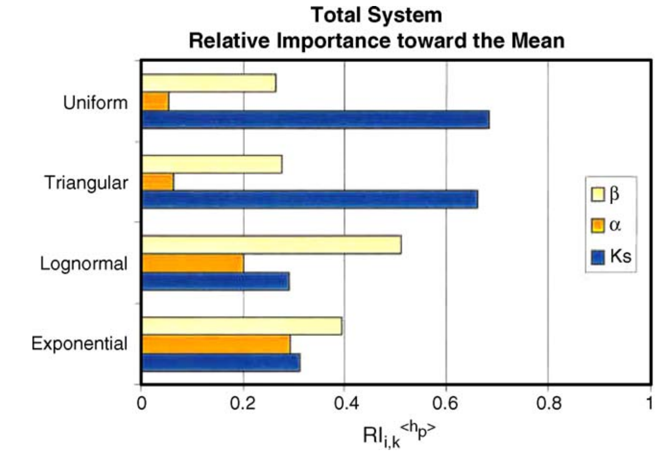

Fig. 4. Total system, $\lambda = 1$ m: relative importance of each parameter for each distribution toward the mean pressure, Eq. (6).

distribution model indicates that limiting the parameters' statistical description to a few moments only is not sufficient for accurate flow prediction but the functional form of the probability distribution needs to be unequivocally resolved.

In order to simplify the depiction of the results for the relative importance of the parameters the detailed mean (and variance) point profiles were averaged over the six layers for each distribution. This lumping of the results over the total system is customarily done in studies of total system performance assessments [14] where the hydraulic behavior is one component of a system's response that may include, geochemical, mechanical, biological and other considerations. Details of the relative importance of the parameters for individual layers are provided in Avanidou [20]. Figs. 4 and 5 plot the relative importance of each parameter toward the mean and the variance of the pressure head for each distribution, Eq. (6). For parameters following a triangular or uniform distribution, uncertainty in K_s is the main contributor to total system uncertainty (about 67% for the mean and between 55% and 67% contribution for the variance), with β being the second dominant (about 27% toward the mean and 30–40% contribution toward the variance) and α being the third

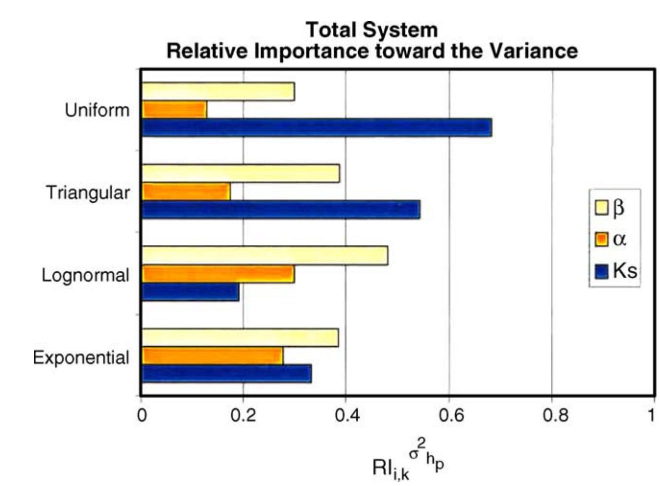

Fig. 5. Total system, $\lambda = 1$ m: relative importance of each parameter for each distribution toward the variance of the pressure, Eq. (6).

factor in terms of importance. For the lognormal and exponential distributions β becomes the most dominant parameter towards both the mean and the variance of the pressure head profile with K_s and α being, approximately, equally important. Overall it appears that for total system analyses the saturated hydraulic conductivity and the pore-size distribution index need to be treated as stochastic parameters with the exact statistical model clearly defined in order to establish the correct hydraulic behavior, whereas α can, perhaps, for the cases of triangular and uniform distributions, be treated as a deterministic parameter. Our results agree with those by Chen et al. [31,32] about the importance of the saturated hydraulic conductivity in unsaturated flow predictions. They also agree with those by Boateng and Cawlfield [24] on the importance of the β parameter but do not support these authors' conclusion that the saturated hydraulic conductivity can be considered as a deterministic variable with no significant effect on the probability outcome. Finally, our results for the lognormal distribution agree with the conclusions by Mishra et al. [33] about the importance of β and K_s rather than α on pressure head. In terms of a site characterization campaign some guidelines that can be advanced based on our results are that there needs to be a clear delineation of the statistical structure of the data, which is not limited to a few moments only but extends to clarification of the exact distribution, and that in the presence of limiting resources emphasis should, perhaps, be given initially to characterization of K_s and β.

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